How stars live and die

The global observed properties of stars:

Total luminosity

Distribution in wave lengths of the emitted radiation

Radius

Mass

Composition

Rotational velocity

Surface temperature

Structure of spectral lines (a huge amount of information)

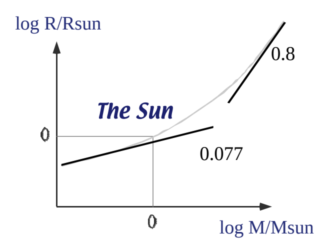
Space velocity

The parameters of the Sun

$$L_{sun} = 3.86 \times 10^{33} erg/s$$

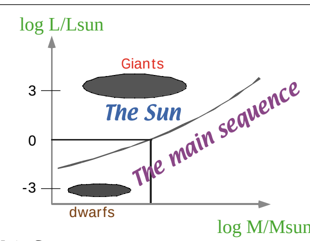
$$M_{sun} = 1.989 \times 10^{33} gm$$

$$R_{sun} = 6.96 \times 10^{10} cm (= 109 R_{Earth})$$



$$R/Rsun = \begin{cases} (M/Msun)^{0.077} & \text{Low masses} \\ (M/Msun)^{0.8} & \text{High masses} \end{cases}$$

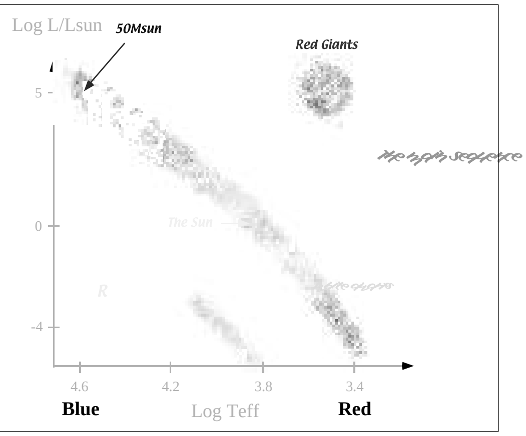
M/R ~ constant for low masses



Along the Main Sequence

$$L/Lsun \begin{cases} (M/Msun)^{5.5} & \text{Low mass} \\ (M/Msun)^{5.2} & \text{High Mass} \end{cases}$$

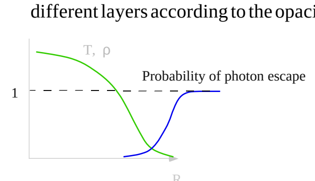
The luminosity increases with a very high power of the mass ~ 5



$$L = \underbrace{4\pi R^2}_{\text{area}} \underbrace{\sigma T_e^4}_{\text{Black body emission}}$$

σ is the Stefan Boltzmann constant

Te is the effective temperature (an equivalent temperature of the surface- stars do not have 'a surface'-the emission is from different layers according to the opacity)



The emission is a composite of many temperatures. Te is an effective value

White Dwarfs parameters

$L=(10^{+2} - 10^{-5})$Lsun **Below this low luminosity it is difficult to detect**

$R=10^{-2}$ Rsun

M=(0.6+/-0.2)Msun

$$\bar{\rho}_{sun} = \frac{M_{sun}}{4\pi R^3/3} = 1.4 gm/cm^3$$

$$\bar{\rho}_{WD} \quad \frac{\bar{\rho}_{sun}}{(R_{WD}/R_{sun})^3} = 10^6 g/cm^3$$

Red giants will float in the air!

Parameters of Red Giants

L ~ 10^5Lsun

Te ~ 3000K

R ~ (100-1000)Rsun

Mean density ~ 10^{-6} - 10^{-9} g/cm3

The state of the matter in stars

The conditions in the Sun are:

$$T = (1-15)\times 10^6 K$$

$$\rho = (10^{-3} - 10^2) g/cc$$

What are the conditions of the matter under such conditions?

All atoms are ionized

Let χ **be the energy required for ionization**

Condition for ionization $kT \sim \chi$

Typical ionization potentials

H=13.6eV, HeI=24eV, HeII=54eV, metals = 2-8eV

For Hydrogen: $T = \frac{\chi(H)}{k} = 1.6\times 10^5 K$

Hydrogen will be ionized for higher temperatures

Actually, ionization takes place at $T = \frac{\chi/10}{k}$

The matter in the Sun (and stars) is in a form of plasma: all atoms are ionized except for few atoms near the surface of the star where the temperatures are the lowest

The particles in the plasma will behave as an ideal gas if:

The average electrostatic interaction $\frac{(Z_1 e)(Z_2 e)}{\langle r \rangle} << kT$ **Kinetic energy Per particle**

Mean distance between particle

But: $\langle r \rangle \quad \rho^{-1/3}$

The condition for an ideal gas is therefore $A\rho^{1/3} << T$

For hydrogen: $A = 2\times 10^5$

The ideal gas approximation is good in the Sun

Electrostatic interactions are important in White Dwarfs

When quantum effects become important?

The uncertainty principle is: $\Delta p \Delta x \approx \hbar$

When the velocity of the particle is v<<c then

$$E = \frac{p^2}{2m} \sim p^2$$

Assume the particle is in a box Δx^3

Then: $\Delta x \sim \rho^{-1/3}$

The uncertainty principle implies that: $E^{1/2}\rho^{-1/3} \sim \hbar$

The energy per particle or energy/mass $E \sim \hbar^2 \rho^{2/3}$

In $E \sim \hbar^2 \rho^{2/3}$

E is the quantum energy associated with the uncertainty principle. When ρ increases E increases.

When: $E \sim \hbar^2 \rho^{2/3} \sim kT$

Quantum effects become important

As ρ increases, E increases because the available space for the particles decreases. As the particles are restricted to a smaller volume, their energy rises.

The condition is: $$\frac{1.2 \times 10^5 \rho^{2/3}}{\mu_e^{2/3}} \leq \mathrm{T}$$

μ_e-molecular weight per electron (for hydrogen μ_e =1)

Quantum effects are negligible in the Sun

Quantum effects are extremely important in White Dwarfs!

How high should T be in White dwarfs for quantum effect to be negligible?

$$\mathrm{T} > 1.2 \times 10^5 (10^6)^{2/3} \sim 1.9 \times 10^9 \mathrm{K}$$

But White dwarfs are much cooler!

The matter at high temperatures

When $E_e \sim m_e c^2$ **(m_e- electron mass)**

we have to bring in special relativity. The condition is:

$$kT \sim m_e c^2 \quad \textbf{or:} \quad T \geq \frac{m_e c^2}{k} = 5.93 \times 10^9 \mathrm{K}$$

When kinetic energy >> rest mass energy ($m_e c^2$) it is possible to neglect the rest mass of the electrons. The electrons behave as particles with no mass - like photons

If the electrons behave like photons then:E=pc

Uncertainty $\longrightarrow$ $\Delta p \Delta x \sim \hbar$

Relativity v~c $\longrightarrow$ $E \sim p$

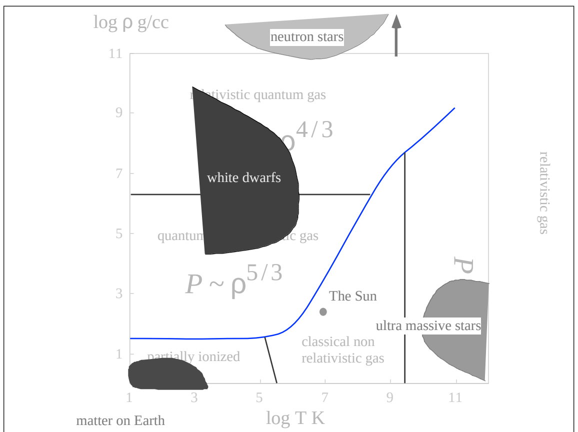
Note: here p is momentum

Hence $\longrightarrow$ $E \sim p \sim \dfrac{\hbar}{\Delta x} \sim \hbar \rho^{1/3}$

or $e = \dfrac{E}{V} = P \sim \rho^{4/3}$

Recall: energy/volume = pressure

When relativity and quantum theory are important the pressure increases as the density to the 4/3 power.



This is the virial theorem

For a star in equilibrium $E_{internal}$=-$AE_{gravitational}$

A=0.5-1

$$\frac{3}{2}kTM = -\frac{1}{2}E_{grav} = \frac{1}{2}\frac{GM^2}{R}$$

As the star contracts, R decreases and T increases.

The contracting star reduces the gravitational energy (it becomes more negative). The difference in gravitational energy is radiated out.

A classical star contracts, radiates the extra energy out and heats up. The star emits energy and heats.

The star has a negative heat capacity.

The quantum star

$$dE_{\text{int}} = \left(\frac{\partial E_{\text{int}}}{\partial T}\right)_{\rho} dT + \left(\frac{\partial E_{\text{int}}}{\partial \rho}\right)_{T} d\rho$$

In ideal gas $\left(\frac{\partial E_{\text{int}}}{\partial \rho}\right)_{T} = 0$

In quantum Electron gas $\left(\frac{\partial E_{\text{int}}}{\partial \rho}\right)_{T} \neq 0$

The contraction of the gas raises the particles to higher energies and consequently, the derivative dos not vanish

The gravitational energy does not depend on T, hence

$$dE_{grav} = -\left(\frac{\partial E_{grav}}{\partial \rho}\right) d\rho$$

But in all stars the internal and gravitational energies are connected, so

$$E_{int} = -E_{grav}$$ **The virial theorem**

Hence:

$$-\left(\frac{\partial E_{grav}}{\partial \rho}\right) d\rho = \left(\frac{\partial E_{int}}{\partial T}\right)_{\rho} dT + \left(\frac{\partial E_{int}}{\partial \rho}\right)_{T} d\rho$$

Eliminate $(\partial T / \partial \rho)_{star}$

$$\left(\frac{\partial T}{\partial \rho}\right)_{star} = -\frac{\dfrac{\partial E_{grav}}{\partial \rho} + \left(\dfrac{\partial E_{int}}{\partial \rho}\right)_{T}^{gas}}{\left(\dfrac{\partial E_{int}}{\partial T}\right)_{\rho}^{gas}}$$

For all gases

This is a thermodynamic law $\left(\frac{\partial E_{int}}{\partial T}\right)_{\rho}^{gas} > 0$

For the gravity $E_{grav} \sim -M^2 R^{-1} \sim -M^2 \rho^{1/3}$

$$\partial E_{grav} / \partial \rho = dE_{grav} / \partial \rho \sim -M^2 \rho^{-2/3}$$

In a star composed of ideal gas:

$$\frac{dT}{d\rho} = -\frac{\dfrac{\partial E_{grav}}{\partial \rho}}{\dfrac{\partial E_{int}}{\partial T}} > 0$$

When the star contracts ρ increases and so does T. The star contracts and heats (the star loses energy, contracts and heats)

Non relativistic quantum gas

$$E_{int} \sim M\rho^{2/3} \qquad \partial E_{int} / \partial \rho \sim M\rho^{-1/3}$$

$$\left.\frac{dT}{d\rho}\right|_{star} \sim \left(\frac{M^2}{\rho^{2/3}} - \frac{M}{\rho^{1/3}}\right) / \underbrace{\left(\frac{\partial E_{int}}{\partial T}\right)}_{>0}$$

Hence $\left.\frac{dT}{d\rho}\right|_{star} \sim \frac{M}{\rho^{2/3}} - \frac{1}{\rho^{1/3}}$

The temperature may change sign

Relativistic quantum gas

$$E_{int} \sim M\rho^{1/3} \qquad (\partial E_{int} / \partial\rho)_T \sim M\rho^{-2/3}$$

$$\left.\frac{dT}{d\rho}\right|_{star} \sim \frac{M}{\rho^{2/3}} - \frac{1}{\rho^{2/3}}$$

In a classical star T always rises as ρ increases (due to contraction)

In a non relativistic quantum star, when ρ is small, the first term dominates, and dT/dρ >0.

When ρ increase, so does the second term, as eventually dT/dρ changes sign due to quantum effects

At first, the star contracts and heats up, but after a certain point the star contracts and cools

Relativistic quantum star

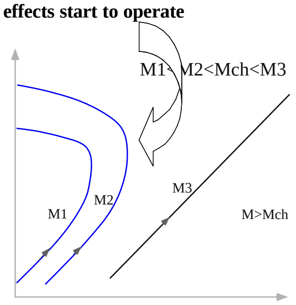
CaseA M<1 (the stellar mass is smaller than ~ 1Msun)

$$\frac{dT}{d\rho} < 0$$ **The star contracts and cools**

CaseB M>1 high mass star

$$\frac{dT}{d\rho} > 0$$ **The star contracts and heats**

The quantum effects start to operate



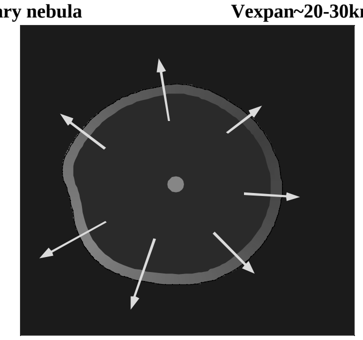
Stellar evolution in the density-temperature plane

Stars with masses < Mch can reach a maximum temperature, hence they cannot ignite all nuclear reactions. After reaching max. T they continue to contract but cool. The cooling stars are called White Dwarfs

Stars with masses > Mch contract and heat all the time. They reach higher and higher temperatures and hence ignite one nuclear reaction after the other.

Observations of the death of light stars

Planetary nebula **Vexpan~20-30km/s**



The nebulae is about 1/2-3 light years across

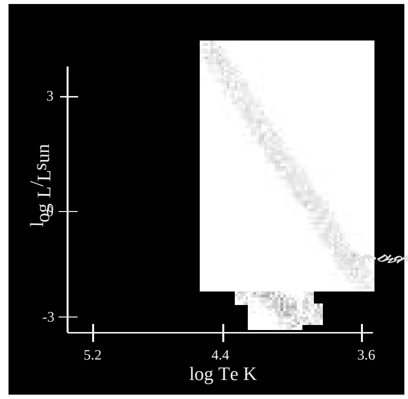
The temperature of the central star 30,000-100,000K

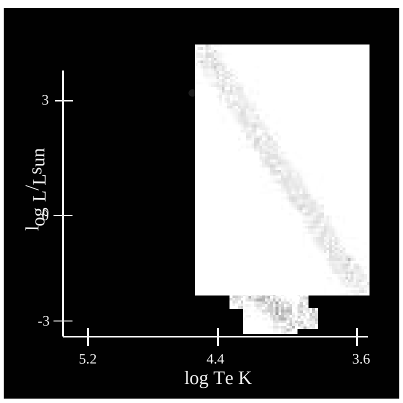
The stellar UV ionizes the expanding nebulae.

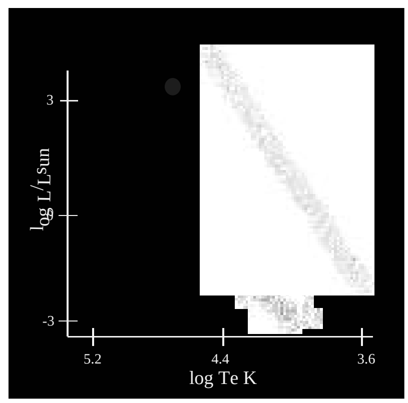
1965 Harman and Seaton: a correlation between the location of the central star on the HR diagram and the size of the nebulae.

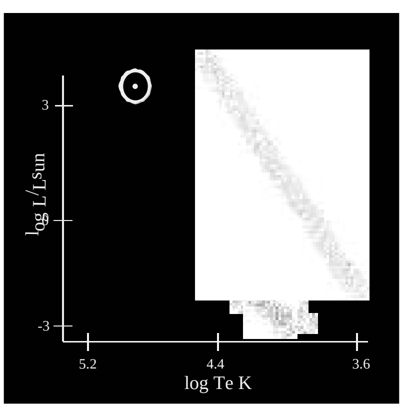
The nebulae expands all the time. The central star first heats at almost constant L and then cools with decreasing luminosity.

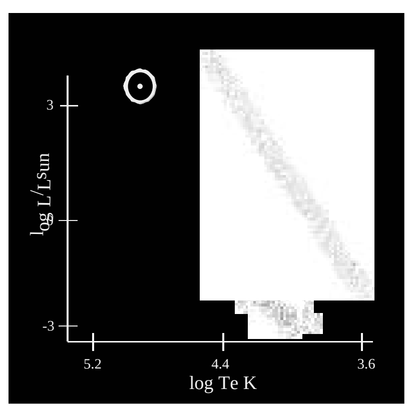
Eventually, the nebulae fades away and the star becomes a White dwarf.

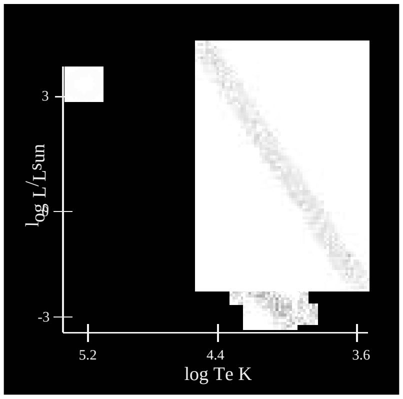


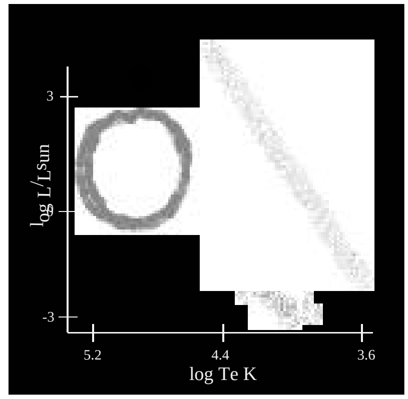









3
0
-3
log L/LSun
5.2
4.4
3.6
log Te K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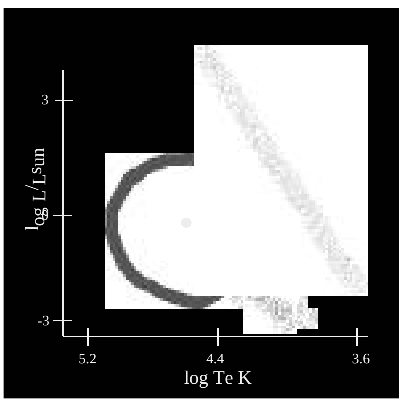
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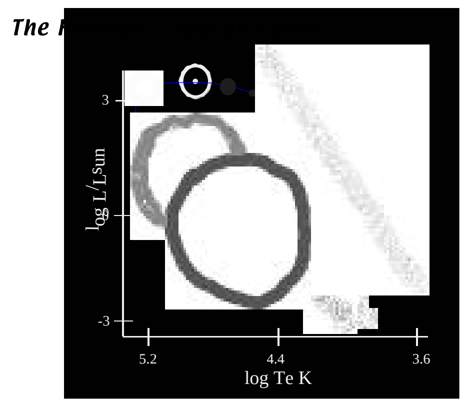
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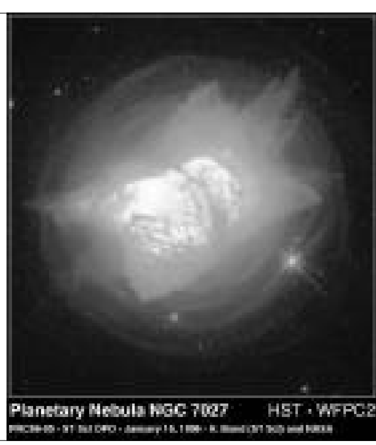
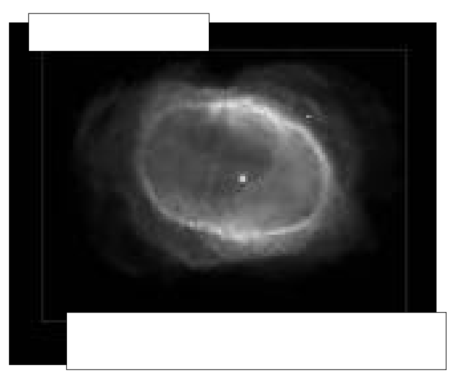
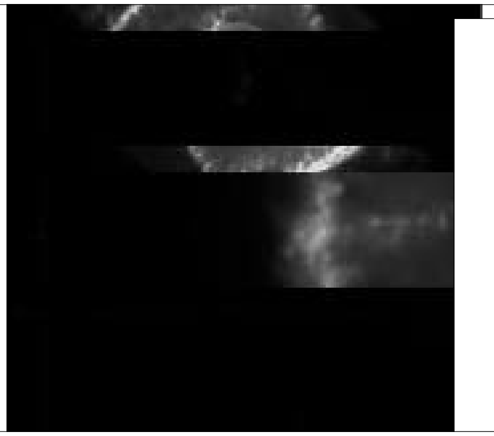
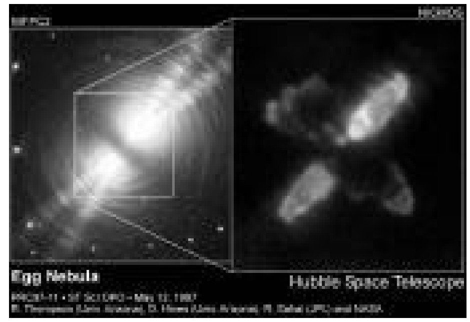
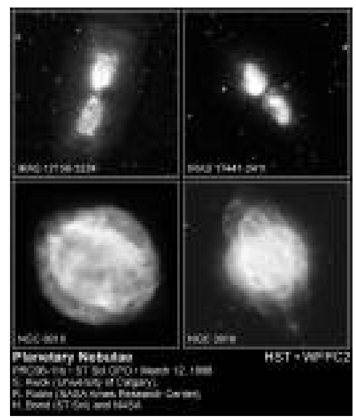

Planetary Nebula NGC 7027
HST · WFPC2

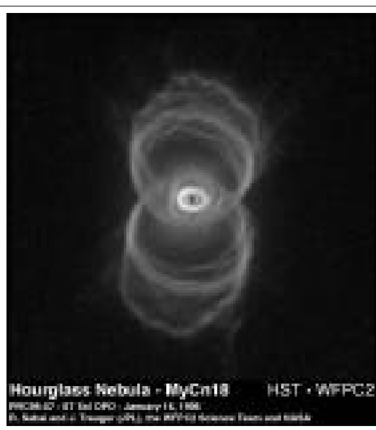


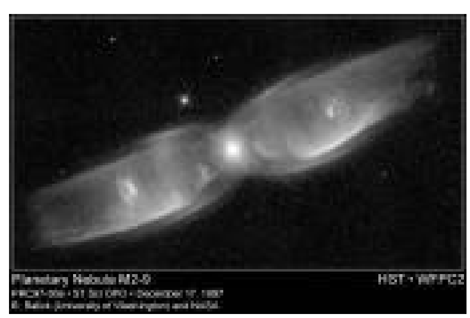


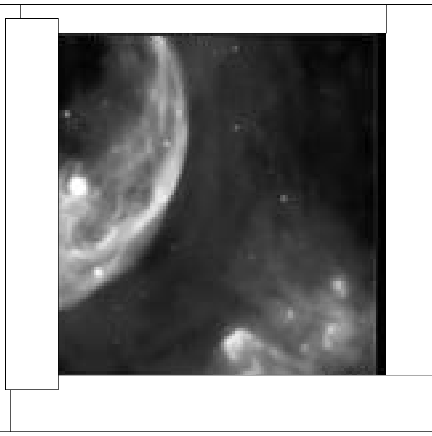

Egg Nebula
Hubble Space Telescope


Henize 1357 • Stingray Nebula
HST • WFPC2


Planetary Nebulae
HST • WFPC2


Hourglass Nebula - MyCn18
HST • WFPC2


Planetary Nebula M2-9
HST • WFPC2



The expansion speed of the nebulae is 20-30km/s and the total mass is 10^{-1}-10^{-2} Msun. When they reach a size of about 3ly, they fade away.

The time needed for the nebulae to fade away is 3-5 × 10^4yr.

This is the time required by the star to heat up, cool and become a white dwarf.

$$\underbrace{h\nu}_{\substack{non-visible\\ stellar UV\\ \lambda<912A}} + H \rightarrow H^+ + e^- - 13.56eV$$

e ejected from ground state

$$H^+ + e^- \rightarrow H + \underbrace{h\nu'}_{\substack{visible\\ photon\\ \lambda>3500A}}$$

e returns to an excited state

Cooling of White Dwarfs

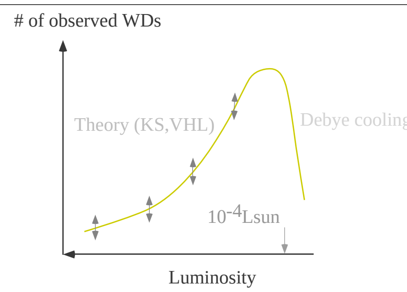
$$\tau_{cool} = \frac{\text{change in energy content}}{\text{luminosity}}$$

Central Temperature K	Luminosity Solar L	Cooling time years
2×10^8	3×10^2	10^5
2×10^7	2×10^{-2}	2×10^8
4×10^6	8×10^{-3}	10^{10}

The age of the Universe is about 15 Billion yrs.

The faintest observed WD has $L \sim 5\times10^{-5} L_{sun}$

The cooling to this time is about the age of the Universe



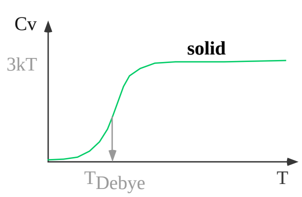
How many WD there are in the Milky Way?

Did the early WDs have time to cool to very low luminosities?

Was the rate of WD formation constant?

The White Dwarfs provide evidence to what happened at the beginning of the Milky Way.

Stellar Evolution Theory claims that stars with masses higher than Mch can become WD's via extensive mass loss which occurs in the Reg Giant phase.



The plasma solidifies at

$$\Gamma = \frac{(Z_1 e)(Z_2 e)}{\langle r \rangle k_B T} \quad 160$$

The heat capacity of the solid is 3kT/mol until T_{Debye}.

Does G change in time?

The long lifetime of WD allows to check if G=G(t)

$$dE_{grav} = \frac{GM^2}{R^2} dR - \frac{GM^2}{R} dG$$

If G decreases in time, dG<0, more gravitational energy is released and the cooling rate decreases.

Present observations yield:

$$\left|\frac{1}{G}\frac{dG}{dt}\right| < 10^{-12} (yr)^{-1}$$

The end of this talk